

Second and higher harmonics generation with memristive systems

Guy Z. Cohen,¹ Yuriy V. Pershin,² and Massimiliano Di Ventra¹

¹*Department of Physics, University of California, San Diego, La Jolla, California 92093-0319*

²*Department of Physics and Astronomy and USC Nanocenter,
University of South Carolina, Columbia, SC, 29208*

We show that memristive systems can be used very efficiently to generate passively both double and higher frequency harmonics. A technique for maximizing the power conversion efficiency into any given harmonic is developed and applied to a single memristive system and memristive bridge circuits. We find much higher rates of power conversion compared to the standard diode bridge, with the memristive bridge more efficient for second and higher harmonics generation compared to the single memristive system. The memristive bridge circuit optimized for second harmonic generation behaves as a two-quarter-wave rectifier.

In nonlinear optics, the phenomenon of second-harmonic generation (SHG), demonstrated in 1961¹, refers to the possibility of creating an outgoing light beam with double the frequency of the impinging beam². This is usually achieved with the help of nonlinear crystals. The smallness of the nonlinearity, however, limits the efficiency of direct SHG. The use of lenses and mirrors improves the efficiency up to 85% by making the light pass repeatedly through a nonlinear medium³. A common application of SHG in optics is the creation of laser sources with modified frequencies.

In the field of electronics, SHG is termed as frequency doubling. Electronic frequency doublers can be divided into active ones, requiring an external power source, and passive ones, operating exclusively on the power of the input signal (see, e.g., Ref. 4). A straightforward realization of a passive frequency doubler is based on a diode bridge⁵, which produces a full-wave rectified output (mathematically, the absolute value of a sinusoidal input). Using Fourier analysis, it can be shown that the diode bridge transforms 4.50% of the input power into the second harmonic, and 18.9% of the input power into all higher harmonics⁵. In addition, by symmetry its output signal contains only even harmonics.

Memristors⁶, which belong to the more general class of memristive systems⁷, are resistive circuit elements with memory. A recent understanding⁸ of the memristive nature of resistance switching in memory cells has attracted a lot of attention to the field of memory elements⁹, which comprises also memcapacitive and meminductive systems¹⁰, namely, capacitors and inductors with memory.

In this paper we demonstrate that a passive circuit with a *single* memristive device generates second and higher harmonics signals on a load resistor with significantly higher efficiency than that of the diode bridge. Moreover, the efficiency of harmonics generation can be further improved employing a *memristive bridge* introduced in this paper. We note that SHG in large memristor networks was anticipated in Ref. 11 but no detailed analysis was given.

We start by showing that memristive systems indeed generate second and higher harmonics using a simple

model of resistance switching introduced in Ref. 8 to describe the response of a thin TiO₂ film of a width D sandwiched between two Pt electrodes. The phenomenon of resistance switching in this system is understood¹² as a field-induced migration of oxygen vacancies changing the width w of a doped low-resistance region. Electrical current in the positive direction (towards the thick line in the circuit symbol of the memristor in Fig. 1(a)) increases w , decreasing the resistance. If the maximum and minimum resistance values, obtained at $w = 0$ and $w = D$, are denoted by R_{off} and R_{on} , respectively, and the charge flow through the device needed to completely switch it from one limiting state into another is q_0 , then the device memristance (memory resistance) can be written as⁸

$$R(w) = R_{\text{off}} + (R_{\text{on}} - R_{\text{off}}) \frac{w}{D}, \quad (1)$$

where $w/D = q/q_0$, and q is the cumulative charge flowing through the device.

We first consider a sine voltage source of period T connected in series to a memristive device $R(w)$ and a load resistor R_1 (Fig. 1(a)). Kirchhoff's voltage law for this circuit is

$$V_0 \sin \omega t - q[R(w) + R_1] = 0, \quad (2)$$

where $R(w)$ is given by Eq. (1) and $\omega \equiv 2\pi/T$. Solving

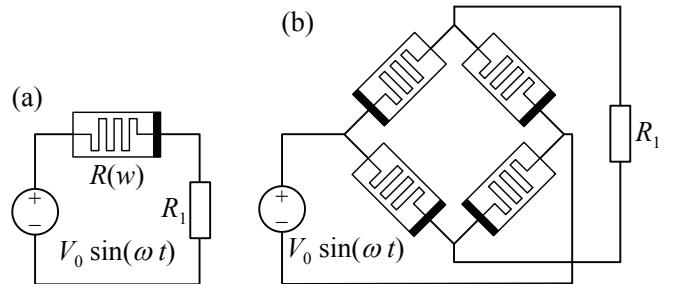


FIG. 1. (a) Circuit consisting of a time-dependent voltage source connected to a memristive system with memristance $R(w)$ and a load resistor R_1 . (b) Memristive bridge circuit.

Eq. (2) with $q(t=0) = 0$, we find the current

$$I(t) = \frac{V_0 \sin(\omega t) / (R_{\text{off}} + R_1)}{\sqrt{1 - \frac{4R_2(R_{\text{off}} - R_{\text{on}})}{(R_{\text{off}} + R_1)^2} \sin^2(\frac{\omega t}{2})}}, \quad (3)$$

where $I(t) \equiv \dot{q}$, and $R_2 \equiv V_0 / (q_0 \omega)$ is a constant with dimensions of resistance. First, we see that the current $I(t)$ in Eq. (3) has the same periodicity as the source and, therefore, can be written as a Fourier series. Second, the numerator in Eq. (3) is a static resistivity current expression, while the denominator contains corrections due to memory. These corrections give rise to all higher harmonics. In many memristive devices $R_{\text{on}} \ll R_{\text{off}}$, and it is reasonable to assume $R_1 < R_{\text{off}}$. Under such assumptions, it follows from Eq. (3) that the condition for considerable generation of higher harmonics is having R_2 / R_{off} of order one (while keeping the denominator real). For a given memristor, the ratio R_2 / R_{off} can be increased either by increasing the amplitude of the source, V_0 , or by decreasing its frequency, $\omega / 2\pi$.

The memristor model given by Eq. (1) combined with $w/D = q/q_0$ is convenient for analytical calculations. However, it does not limit $R(w)$ between R_{on} and R_{off} . In order to obtain quantitative results, we then suggest a more realistic model consisting of Eq. (1) and

$$\dot{w} = \frac{D}{q_0} [\theta(1 - w/D)\theta(\dot{q}) + \theta(w/D)\theta(-\dot{q})] \dot{q}, \quad (4)$$

where the θ functions in Eq. (4) constrain w to satisfy $0 \leq w \leq D$, in agreement with experimental data^{13,14}.

It follows from Eqs. (4) and (1) that \dot{w} has the periodicity of \dot{q} , and that the current $I(t)$ has the same periodicity as that of the voltage source. However, $I(t)$ cannot be found in a closed analytical form now, and we solve the problem numerically. If we denote the average power dissipated on the load resistor in the i -th harmonic as P_i and the average power of the source as P_{source} , three important ratios come to mind: P_2/P_{source} , P_3/P_{source} and $(\sum_{k=2}^{\infty} P_k)/P_{\text{source}}$, which are the second, third and higher harmonics conversion efficiencies, respectively. Identifying the system parameters maximizing these ratios will enable us to generate, upon application of a suitable band pass filter at the output, the desired harmonics passively and with minimal losses.

Considering all possible parameters, we can write the general functional dependence of the above ratios as

$$\frac{P_k}{P_{\text{source}}} = f_k(\omega, V_0, q_0, w(t=0), R_{\text{off}}, R_{\text{on}}, R_1), \quad (5)$$

$$\frac{\sum_{k=2}^{\infty} P_k}{P_{\text{source}}} = g(\omega, V_0, q_0, w(t=0), R_{\text{off}}, R_{\text{on}}, R_1). \quad (6)$$

As noted above, these ratios cannot be written in closed analytical form. Moreover, the numerical maximization is difficult due to the large number of parameters. It is shown below that it is reasonable to select $w(t=0) = 0$. The number of parameters can be further reduced utilizing the Buckingham π theorem of dimensional analysis¹⁵.

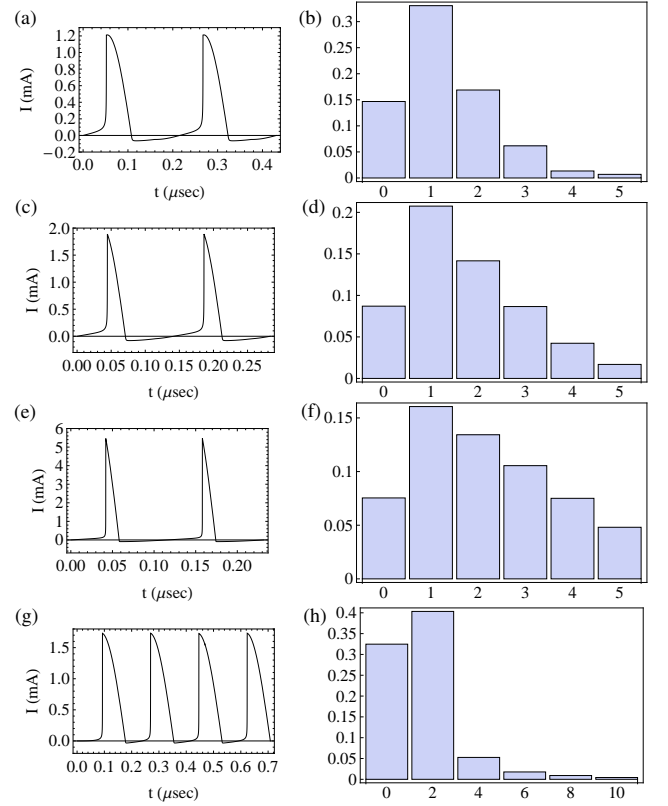


FIG. 2. (color online) Current $I(t)$ and corresponding power ratio distribution (P_i/P_{source}) after maximization of the power conversion into the second (a,b), third (c,d), and all higher harmonics (e,f) in the single memristive device circuit and into the second harmonic in the memristive bridge circuit (g,h). $I(t)$ has been obtained using $V_0 = 1\text{V}$, $q_0 = 3 \cdot 10^{-12}\text{C}$ and $R_{\text{off}} = 20\text{k}\Omega$.

Its application results in functions of dimensionless variables for the above ratios:

$$\frac{P_k}{P_{\text{source}}} = \tilde{f}_k \left(\frac{R_1}{R_{\text{off}}}, \frac{R_2}{R_{\text{off}}}, \frac{R_{\text{off}}}{R_{\text{on}}} \right), \quad (7)$$

$$\frac{\sum_{k=2}^{\infty} P_k}{P_{\text{source}}} = \tilde{g} \left(\frac{R_1}{R_{\text{off}}}, \frac{R_2}{R_{\text{off}}}, \frac{R_{\text{off}}}{R_{\text{on}}} \right). \quad (8)$$

The left-hand sides of Eqs. (7) and (8) are maximized numerically considering the following realistic ranges of dimensionless parameters⁸: $10^{-5} \leq R_1/R_{\text{off}} \leq 10$, $10^{-4} \leq R_2/R_{\text{off}} \leq 10$ and $10 \leq R_{\text{off}}/R_{\text{on}} \leq 10^3$. We find that the maxima of P_2/P_{source} and P_3/P_{source} depend weakly on $R_{\text{off}}/R_{\text{on}}$ and lie in the range $150 < R_{\text{off}}/R_{\text{on}} < 1000$. Consequently, we set $R_{\text{off}}/R_{\text{on}} = 200$. For $\sum_{k=2}^{\infty} P_k/P_{\text{source}}$ it is found that $R_{\text{off}}/R_{\text{on}} = 1000$ gives the best value.

Table I presents conversion efficiencies and lists the optimal parameter values. Clearly, in all cases the single memristive device circuit provides significantly higher conversion rates than the diode bridge. Figs. 2(a)-(f) show the current through the load resistor and corresponding harmonics power spectrum. Since the memristive device is a *dynamically adaptive system*, $I(t)$ is very different from the sine shape of the source.

Optimized quantity	$\frac{R_1}{R_{\text{off}}}$	$\frac{R_2}{R_{\text{off}}}$	$\frac{R_{\text{off}}}{R_{\text{on}}}$	Rate, 4 diodes	Rate, 1 memr.	Rate, 4 memr.
P_2/P_{source}	0.0362	1.71	200	4.50%	16.9%	40.3%
P_3/P_{source}	0.0194	1.13	200	0%	8.66%	0%
$\sum_{k=2}^{\infty} P_k / P_{\text{source}}$	0.00598	0.924	1000	18.9%	48.6%	56.4%

TABLE I. Optimized parameter values for single memristive device circuit, power conversion rates for diode bridge (Rate, 4 diodes), optimized power conversion rates for single memristive device (Rate, 1 memr.) and memristive bridge (Rate, 4 memr.) circuits.

It is also important to check how the initial value of $w(t = 0)$ affects the higher harmonics generation. We have performed extensive numerical simulations and found that the power conversion rates (Eqs. (5) and (6)) always increase with a decrease of w . This observation justifies our choice of $w(t = 0) = 0$ corresponding to $R(w(t = 0)) = R_{\text{off}}$. Moreover, we note that once a certain $R(w(t = 0))$ is selected, the value of R_{off} becomes not important if $R(w(t)) > R_{\text{on}}$. In order to prove this statement, we notice that the load current has the same period as the source, which implies that the memristance also has such periodicity. The memristance increases/decreases with negative/positive current, which changes sign only at $t = nT$ and $(n + 1/2)T$, where n is an integer. Since the memristance decreases immediately after $t = 0$, we conclude the memristance has maxima at $t = nT$, minima at $(n + 1/2)T$ and no other extrema. Therefore, $R_{\text{on}} < R(w) \leq R(w = 0)$ during the entire period, implying that having $w_1 \equiv w(t = 0) > 0$ and off resistance R_{off} is equivalent to having $w(t = 0) = 0$ and off resistance $R(w_1)$. The decrease in the memristance range reduces the circuit nonlinearity, translating into lower efficiency of higher harmonics generation.

The efficiency of the second and higher harmonics generation can be increased even further by utilizing a memristive bridge having the geometry of the diode bridge (Fig. 1(b)). The memristive bridge consists of four identical memristive devices described by Eqs. (1) and (4) rectifying the input signal via the delayed switching effect¹⁶. This rectification reduces the load current periodicity to $T/2$, leaving only the even harmonics. Conse-

quently, the second and higher harmonic generation efficiencies are improved relative to those of the single device circuit, while third harmonic generation is excluded.

In complete analogy with the single device circuit, dimensional analysis for the memristive bridge leads to Eqs. (7) and (8) for the efficiency of harmonics generation. Dimensionless parameters maximizing P_2/P_{source} are $R_2/R_{\text{off}} = 2.82$, $R_1/R_{\text{off}} = 0.0267$ and $R_{\text{off}}/R_{\text{on}} = 1000$ giving $P_2/P_{\text{source}} = 40.3\%$, a substantial improvement over the single device circuit, with same initial conditions. $I(t)$ and the power distribution of harmonics for the memristive bridge are given in Figs. 2(g,h). It is found that with an increase in $R_{\text{off}}/R_{\text{on}}$, the optimized current shape tends towards $|\sin \omega t|$ in even quarters of each period and zero in odd ones. Such a signal has $P_2/P_{\text{source}} = 45.0\%$, which is indeed the asymptotic efficiency at $R_{\text{off}}/R_{\text{on}} \rightarrow \infty$. We recognize this signal shape as being *two-quarter-wave rectified*. For higher harmonics generation, we find the value after optimization of $(\sum_{k=2}^{\infty} P_k)/P_{\text{source}}$ to be 56.4%, a modest improvement over the single device circuit.

In conclusion, we have demonstrated the potential of memristive systems for passive second and higher harmonics generation. Using an approach to maximize the rate of power conversion for a specific harmonic, we have shown that memristive circuits are much more efficient for harmonic generation purposes (at optimal operation conditions) than the traditional diode bridge. In addition, the operation voltage in memristive circuits can be lower than that used in diode circuits because of $\approx 0.7\text{V}$ barrier voltage of silicon p-n junctions⁵. We also anticipate that memristive devices can be beneficially used in harmonics generation in combination with active circuits. An example of such a circuit is a memristive bridge operating with a high resistance load followed by an operational amplifier. Finally, memcapacitive and meminductive systems^{9,10} can be used for passive (and low-dissipative) higher harmonics generation instead of memristive ones. The results of our investigation can be readily tested experimentally.

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